7.1: Functions of Several Variables

When you finish your homework you should be able to…

- Define functions of two or more variables
- Find the domain of a function with two or more variables
- Applications
  - Find a company's cost function
  - Evaluate a Cobb-Douglas function
  - General Applications

Up to this point, we have been mostly working with functions in one variable. In our world we deal situations which have related quantities, such as windchill and cost of a car loan. What variables do you think windchill utilizes?

1. temperature
2. wind speed

How about car loans?

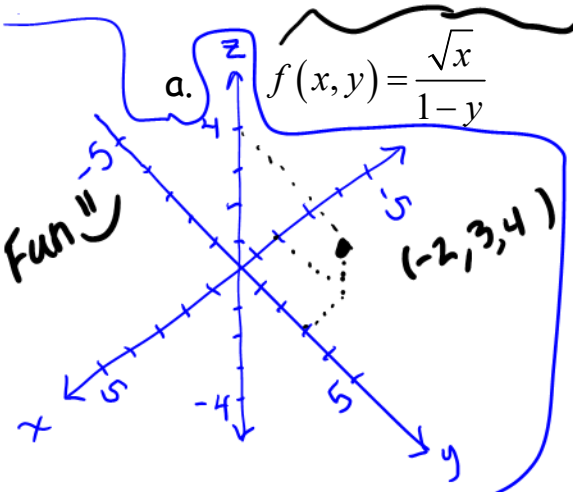
1. interest rate
2. term length
3. principal

FUNCTION OF TWO VARIABLES

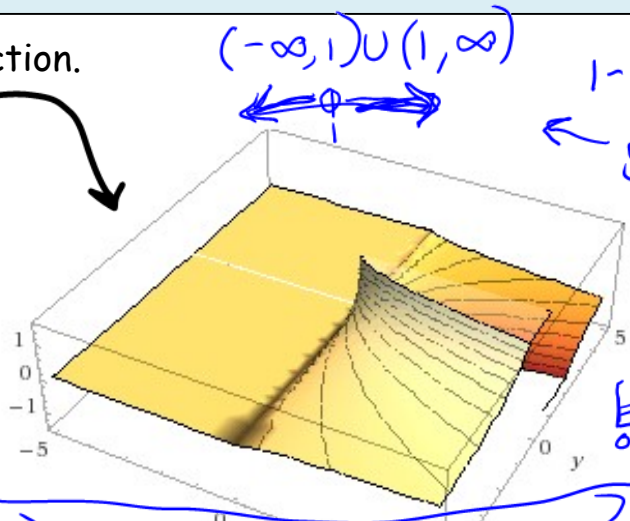
A function f of 2 variables is a rule such that to each ordered pair (x,y) in the domain of f there corresponds one and only one number f(x,y).

Example 1: Find the domain of the function.

a. $f(x,y) = \dfrac{\sqrt{x}}{1-y}$



Fun!!



$(-\infty,1)\cup(1,\infty)$

$1-y \neq 0$

$y \neq 1$

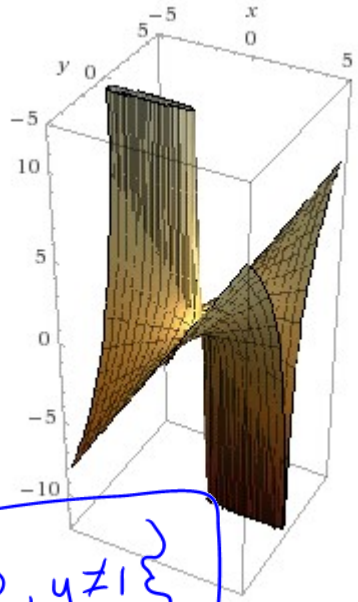
$x \geq 0$

$[0,\infty)$

$\{(x,y): x,y \in \mathbb{R}, x \geq 0, y \neq 1\}$

format — describes limitations

b. $f(x,y) = \dfrac{x}{\ln y}$



$y > 0$ and $y \neq 1$

0 1

$x \in \mathbb{R}$

$\{(x,y): x,y \in \mathbb{R}, y > 0, y \neq 1\}$

c. $f(x,y,z)=\dfrac{e^{1/y}\ln x}{z}$

$x>0$

$y\neq 0$

$z\neq 0$

$\{(x,y,z): x,y,z\in\mathbb{R}, x>0, y\neq 0, z\neq 0\}$

Example 2: Evaluate the function at the given ordered pair of ordered triple.

a. $f(x,y)=xe^{y}+ye^{x}$, find $f(-1,1)$

x y

$f(-1,1)=(-1)e^{1}+1e^{-1}$

$f(-1,1)=-e+\dfrac{1}{e}$

$f(-1,1)=\dfrac{-e^2+1}{e}$

So $\left(-1,1,\dfrac{-e^2+1}{e}\right)$ is a point on the graph of f.

b. $g(x,y)=\ln\left(x^3-y^2\right)$, find $g(e,0)$

$g(e,0)=\ln\left[(e)^3-(0)^2\right]$

$=\ln e^3$

$=\boxed{3}$

$\ln e^3 = 3\ln e$

So $(e,0,3)$ is a point on the graph of g.

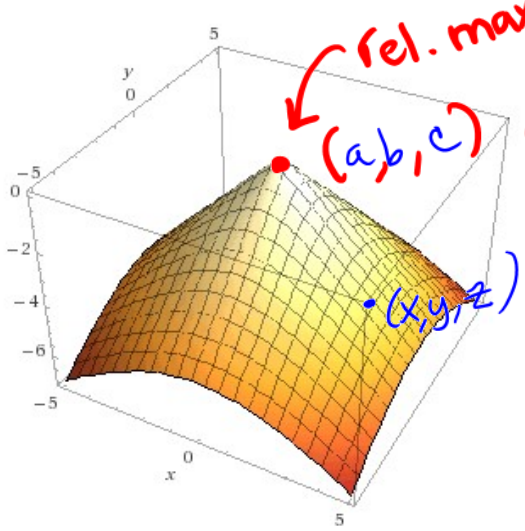
c. $f(x,y,z)=z\sqrt{x}\ln(y)$, find $f(4,e,-1)$

$f(4,e,-1)=(-1)\sqrt{4}\ln e$

$f(4,e,-1)=-2(1)$

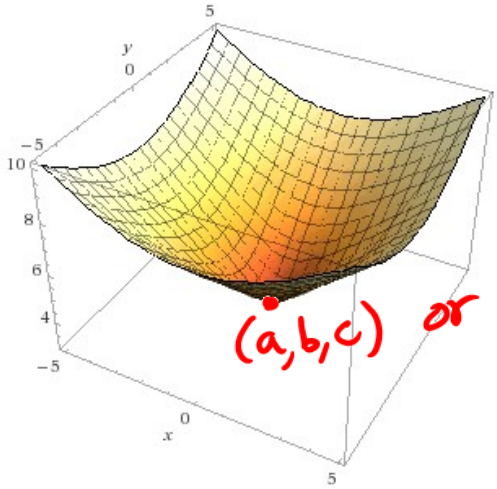
$f(4,e,-1)=\boxed{-2}$

So $(4,e,-1,-2)$ is a point on the graph of f.



RELATIVE MAXIMUM POINT

A point ~~$f(4,e,-1)$~~ (a,b,c) on a surface ~~$f(4,e,-1)$~~ $z=f(x,y)$ is a relative maximum point if $f(a,b) \geq f(x,y)$ for all (x,y) in some region surrounding (a,b).



RELATIVE MINIMUM POINT

A point ~~$f(4,e,-1)$~~ (a,b,c) on a surface ~~$f(4,e,-1)$~~ $z=f(x,y)$ is a relative minimum point if $f(a,b) \leq f(x,y)$ for all (x,y) in some region surrounding (a,b).

APPLICATIONS

Example 3: Finding a company's cost function.

A company manufactures surfboards and stand up paddleboards (SUPs). It costs $60 to make each surfboard, $80 to make each SUP, and fixed costs are $1000.

a. Find the cost function.

x is the # of surfboards produced

y " " " " SUPs

$$C(x, y) = 60x + 80y + 1000$$

b. Use the cost function to find the cost of producing 10 surfboards and 20 SUPs.

$$C(10, 20) = 60(10) + 80(20) + 1000$$

$$C(10, 20) = 3200 \text{ (dollars)}$$

Evaluating a Cobb-Douglas Production Function

A function used to model the output of a company or nation is called a

_____ function, and the most famous is the

_____ production function:

The function expresses the _____ production ___ as a function of

___, the number of units of _____, and _____, the number of units of

_____. Labor is measured in _____, and capital

means _____ capital.

Example 4: Evaluating a Cobb-Douglas Production Function.

At one time, the production of American manufacturing was estimated to be $P = P(L,K) = 2.39L^{0.76}K^{0.24}$. Find $P(2500,450)$.

Example 5: Car Loan

Shannon is buying a 50th Anniversary Edition Ford Mustang GT. She wants to have as low a payment as possible. She is offered two options. The first option is a loan amount of $40,000 at 2.1% interest over 60 months. The second option has the same loan amount at 3.9% interest over 72 months. Which plan should she use? Why? Use the following function:

$$P = P(A, i, N) = \frac{iA}{1-(1+i)^{-N}}$$

where P represents principal, i represents interest, and N is the number of payments.

A = payment amount

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